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Single and double ionization of lead by electron impact

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Abstract. Theoretical calculations of electron impact single and double ionization cross-sections for ground state lead atoms have been performed in the binary encounter approximation (BEA) in the energy region ranging from respective near thresholds to 3000 eV. The accurate expression for $\sigma_{\Delta E}$ (cross-section for energy transfer ΔE) including exchange and interference as given by Vriens and Hartree-Fock velocity distributions for the target electrons have been used throughout the calculations. It is concluded that beyond 10.64 eV impact energy single ionization cross-sections are well explained by considering ionization of $6p$ and $6s$ shells only. The direct double ionization cross-sections obtained theoretically cannot explain the recent experimental observations. Inclusion of contributions of the Auger effect due to vacancy in $5d$ and $5p$ shells brings the results of double ionization cross-sections in reasonably good agreement with the experimental data. The identification of the shells whose ionization leads to the Auger effect contributing to double ionization is a remarkable aspect of the present investigation.

PACS. 34.80.Dp Atomic excitation and ionization by electron impact

1 Introduction

Absolute cross-sections for electron impact single and multiple ionization of atoms/ions are of considerable interest in many fields of research ranging from controlled nuclear fusion to astrophysics. In context of applications, ionization processes are important in moderate and high temperature plasmas and in all gaseous environments with an abundance of energetic electrons [1]. Triple and higher multiple ionization processes start at increasing higher energies and the corresponding cross-sections are much smaller in magnitude compared to double ionization cross-sections. Among different multiple ionization processes the double ionization is the most important as the main contribution to the total ionization of the target is given by single and double ionization processes and hence experimental and theoretical studies of these processes are considered to be valuable.

In case of metals there are several difficulties in determination of ionization cross-sections. Firstly, high temperatures are required for most metal atoms to form their vapours. Secondly, in order to obtain numerical magnitudes of cross-sections, absolute number densities in vapour phase are needed. Therefore, ionization cross-sections for metals have been measured only by very few experimental groups and for a limited number of elements [2]. Recently McCartney et al. [3] of the Belfast group have carried out accurate experimental measurements of electron impact single and multiple ionization cross-sections of ground state lead atoms using a pulsed crossed-beam method incorporating time-of-flight spectroscopy at energies ranging from respective near thresholds to 3000 eV. This experiment arranged a pulsed beam of electrons to intersect a thermal beam of ground state lead atoms derived from an oven source. These measurements are important because accurate cross-sections for the process of multiple ionization of heavy metal atoms by electron impact are needed for the accurate modelling of both astrophysical and fusion plasmas.

McCartney et al. [3] observed some interesting features associated with electron impact multiple ionization cross-sections of lead. At high-energy limit of 3000 eV the cross-sections have been found to be decreasing by much less than an order of magnitude as n increases. Usually single and direct double ionization cross-sections of atoms/ions are found to show faster decrease after attaining the maximum value (see McCallion et al. [4] and Syage [5]) and at high impact energy they differ by about an order of magnitude. In experiments on Fe, Cu and Ga (see Shah et al. [6], Bolorizadeh et al. [7] and Patton et al. [8]) the double ionization cross-sections show relatively slow decrease in their values with increase in energy. Interestingly in case of lead it has been seen that σ_1 and σ_2 tend to converge at higher energies and at 3000 eV the cross-section ratio σ_2/σ_1 is found to be 0.81 (see McCartney et al. [3]). This indicates large contributions to double

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ionization cross-sections of lead from Auger processes as direct double ionization cross-sections are not expected to follow the above mentioned trend. McCartney et al. have recognised that in case of lead the removal of an inner shell electron leads to a large number of states whose energy exceeds the threshold for double ionization. However, they have not mentioned about the contributions of Auger effect to double ionization cross-sections from vacancies in specified shells of lead atom. They have compared their experimental data on single ionization with empirical calculations of Lotz [9] and semi-classical calculations of Margreiter et al. [10] in the energy range $15-200$ eV only. The double ionization cross-sections have been compared with the results obtained by a scaling law proposed by Fisher et al. [11]. This situation has arisen due to non-availability of suitable theoretical calculations.

Theoretical calculations of electron impact double ionization cross-sections are considered to be valuable because contributions from different physical processes viz. simultaneous ejection of two electrons, inner shell ionization followed by Auger emission, resonant excitation-autoionization process etc. can be separately estimated at different impact energies. Rigorous theoretical calculation of the integrated double ionization cross-section becomes extremely difficult as it is related with a 4-body Coulomb problem in the final channel and hence such calculations are not available in literature. A few attempts have been made to calculate electron impact double ionization cross-sections for light targets e.g. H^-, He and Li^+ in the Born approximation (see Tweed [12,13] and McGuire [14]). Due to this reason semi-empirical and semi-classical approaches have emerged for calculation of double ionization cross-sections [11,15–17]. Recently Gryzinski and Kunc [18] made use of classical binary encounter approximation in a "statistical way" to derive general analytical expression for calculations of electron impact double ionization cross-section with atomic number $Z \geq 20$ and s or d outer shells with two electrons. It may be noted that the semi-empirical, semi-classical approaches and calculations of Gryzinski and Kunc do not make use of wave functions which are characteristics of the target atom.

In the past the binary encounter approximation (BEA) has been used successfully to calculate electron impact single and double ionization cross-sections for several atoms. A brief discussion on the applications of the BEA to electron impact ionization processes has been given by Jha and Roy [19]. In spite of certain unrealistic features in Gryzinski's [20] mathematical formulation for the process of double ionization the idea of the two double binary encounter processes has physical justification (see Roy and Rai [21]). These processes in fact correspond to the existence of correlation between the electrons of an atom and to the finite probability of the second Born process (see Vriens [22]). Roy and Rai [21] modified the mathematical framework of Gryzinski's theory of electron impact double ionization suitably and introduced necessary corrections. Later on this modified model for calculation of direct double ionization cross-sections was used in case of several atomic/ionic targets incorporating contributions to double ionization from indirect physical processes [23,24] and encouraging results were obtained in all the cases. Gryzinski and Kunc have appreciated the works of Roy and co-workers [23,25]. Recently Jha [26] has calculated electron impact double ionization cross-sections for singly charged positive ions using Hartree-Fock (HF) wave functions for the target electrons. Contributions of ionization-autoionization have been included in these calculations and satisfactory results have been obtained. Here we would like to mention that correlation plays an important role in double ionization process. In this context it may be noted that calculations using correlated wave functions become very complicated particularly for heavier targets. Jha et al. [25] have discussed that the study of double ionization in Gryzinski's double binary encounter model using Hartree-Fock wave functions for the target electrons takes into account the effect of correlation to some extent and hence such studies may be considered to be reasonable (see also Jha and Roy [19]).

Very recently Jha and Roy [19] have reported calculations of electron impact single and double ionization cross-sections for magnesium in the binary encounter model using accurate expression of $\sigma_{\Delta E}$ (cross-section for energy transfer ΔE) including exchange and interference as given by Vriens [27] and HF velocity distributions for the target electrons throughout the calculations. It has been found that electron impact single ionization cross-sections of magnesium are well explained by considering ionization of $3s$ shell only. At the same time it has been observed that inclusion of contributions of Auger effect to double ionization cross-sections brings the calculated results in reasonably good agreement with the experimental observations. The viewpoint of Peach [28] and Boivin and Srivastava [2] that a vacancy in the $2p$ shell of magnesium leads to double ionization has been substantiated theoretically by these calculations. Encouraged by the success achieved in the above mentioned calculations we have considered it worthwhile to take up calculations of single and double ionization cross-sections for lead in the BEA in order to analyse the direct double ionization cross-sections and to identify the contributions to double ionization from Auger effect resulting from vacancies of electrons of different shells.

2 Theoretical methods

We have used the accurate expression for $\sigma_{\Delta E}$ including exchange and interference as given by Vriens [27] for calculating electron impact single ionization cross-sections. The expression used in the calculation has been discussed in detail by Roy and Rai [29] (see also Jha and Roy [19]). A brief presentation of the expression in final form used in the calculations is given below. Using dimensionless variables introduced by Catlow and McDowell [30], the expression for cross-section for a particular incident energy and a particular velocity of the bound electron can be written in the form

$$Q^i(s,t) = \frac{4}{(s^2+t^2+1)u^2} \times \left[\frac{s^2-1}{s^2} + \frac{2t^2}{3}\left(\frac{s^4-1}{s^4}\right) - \frac{\phi \ln s^2}{(s^2+1)}\right](\pi a_0^2)$$

$$\text{where} \quad \phi = \cos\left\{\left(\frac{1}{s^2u+u}\right)^{1/2} \ln s^2\right\}. \qquad (1)$$

Numerical integration of the expression for $Q^i(s,t)$ has been carried out over Hartree-Fock velocity distribution of the bound electron to obtain the ionization cross-section. Thus the expression for electron impact single ionization cross-section for a particular shell of the target is given by

$$Q^i(s) = n_e \int_0^\infty Q^i(s,t) f(t) u^{\frac{1}{2}} dt \qquad (2)$$

The method of calculating electron impact double ionization cross-sections of atoms in double binary encounter model has been discussed in detail in earlier publications [21,23] (see also Jha and Roy [19]). However, it is desirable to give a brief discussion of the expressions which have been used in the present calculations. Electron impact double ionization cross-section including contribution from Auger emission can be written as

$$Q^{ii}(T) = Q^{ii}_D + Q^{ii}_A \qquad (3)$$

where Q^{ii}_D denotes the contribution from direct ejection of the two electrons and Q^{ii}_A that from Auger emission. The expressions for cross-sections corresponding to the two processes of the double binary encounter model leading to direct double ionization are given by (see Jha and Roy [19])

$$Q^{ii}_{sc} = \frac{n_e(n_e-1)}{4\pi\bar{r}^2} \int\limits_{t=0}^{\infty} \int\limits_{U_i}^{E_q-U_{ii}} \sigma_{\Delta E} \times \left[\int\limits_{t=0}^{\infty} \int\limits_{U_{ii}}^{E_q-\Delta E} \sigma_{\Delta E'} f(t)\, U_{ii}^{1/2}\, d(\Delta E')\, dt\right] \times f(t)\, U_i^{1/2}\, d(\Delta E)\, dt \times 8.797 \times 10^{-17} (\pi a_0^2) \qquad (4)$$

and

$$Q^{ii}_{ej} = \frac{n_e(n_e-1)}{4\pi\bar{r}^2} \int\limits_{t=0}^{\infty} \int\limits_{U_i+U_{ii}}^{E_q} \sigma_{\Delta E} \times \left[\int\limits_{t=0}^{\infty} \int\limits_{U_{ii}}^{\Delta E-U_i} \sigma_{\Delta E'}\, f(t)\, U_{ii}^{1/2}\, d(\Delta E')\, dt\right] \times f(t)\, U_i^{1/2}\, d(\Delta E)\, dt \times 8.797 \times 10^{-17}\, (\pi a_0^2). \qquad (5)$$

As in the case of single ionization, we have used the accurate expression for $\sigma_{\Delta E}$ as given by Vriens [27] in the above expressions also. Using dimensionless variables introduced by Catlow and McDowell [30] $\sigma_{\Delta E}$ is given by (see Kumar and Roy [31])

$$\sigma_{\Delta E} = \frac{2}{(s^2+t^2+1)\,u}\left[\left(\frac{1}{\Delta E^2} + \frac{4t^2u}{3\Delta E^3}\right) + \left(\frac{1}{(s^2u+u-\Delta E)^2} + \frac{4t^2u}{3\,(s^2u+u-\Delta E)^3}\right) - \frac{\phi}{\Delta E\,(s^2u+u-\Delta E)}\right]$$

$$\text{where} \quad \phi = \cos\left\{\left(\frac{1}{s^2u+u}\right)^{1/2} \ln s^2\right\}. \qquad (6)$$

Due to indistinguishability of electrons in the symmetrical model of Vriens the cross-sections corresponding to the two processes are exactly equal at all incident energies (see Kumar and Roy [31]) and hence in order to obtain the direct double ionization cross-section, either of the cross-sections should be multiplied by two. In equation (4) u and s^2 have been replaced by U_i and E_q/U_i in the expression for $\sigma_{\Delta E}$ and by U_{ii} and $(E_q-\Delta E)/U_{ii}$ in case of $\sigma_{\Delta E'}$. The only difference in the equation (5) is that s^2 assumes the value $(\Delta E - U_i)/U_{ii}$ in expression for $\sigma_{\Delta E'}$. The function $f(t)$ appearing in equations (2), (4) and (5) is the momentum distribution function (see Catlow and McDowell [30] and Jha and Roy [19]). In case of double ionization $f(t)$ has been constructed replacing u by U_i and U_{ii} for ejection of the first and the second electron respectively. In order to obtain Q^{ii}_A (contribution to double ionization from Auger emission), the single ionization cross-section should be multiplied by Auger yield of the shell under consideration.

We have considered total cross-section for electron impact direct double ionization of lead as given by

$$Q^{ii}_D = Q^{ii}_D(6p,6p) + Q^{ii}_D(6p,6s) + Q^{ii}_D(6p,5d) + Q^{ii}_D(6p,5p)$$

where $Q^{ii}_D(6p,6s)$ stands for the double ionization cross-section corresponding to one electron ejected from the $6p$ shell and the other from the $6s$ shell. The factor $n_e(n_e-1)/4\pi\bar{r}^2$ has been suitably modified for considering the modes of ionization in which the electrons are ejected from different shells. $n_e(n_e-1)$ has been replaced by $n_e1 \times n_e2$ where these two stand for number of electrons in shells under consideration. In order to obtain the value of $\bar{r}$, the atomic radius has been replaced by the mean of the expectation values of radii of the shells (see Jha and Roy [19]). For binding energies we have used the magnitudes of orbital energies of the shells of Pb and Pb^+ as given by Mclean and Mclean [32] in the present calculations. The expectation values of radii reported by Desclaux [33] have been used as shell radii. Hartree-Fock radial wave functions given by Mclean and Mclean [32] have been used to construct momentum distribution functions for the target electrons.

Table 1. Electron impact single ionization cross-sections of Pb in units of 10^{-16} cm^2.

Energy (eV)	Contributions of 6p shell	Contributions of 6s shell	Total	Experiment [3]
7.56	1.63	-	1.63	0.17
10.64	5.39	-	5.39	2.74
15.94	7.11	1.41	8.52	6.00
20.74	7.33	2.20	9.53	7.61
24.44	7.24	2.48	9.72	7.32
39.74	6.35	2.68	9.03	6.80
49.44	5.79	2.58	8.37	6.48
67.54	4.96	2.33	7.29	6.00
100.74	3.94	1.93	5.87	5.18
209.54	2.42	1.20	3.62	3.25
399.44	1.49	0.73	2.22	2.12
487.44	1.26	0.62	1.88	1.80
589.44	1.08	0.53	1.61	1.50
709.44	0.92	0.45	1.37	1.33
859.44	0.78	0.38	1.16	1.14
1045	0.66	0.32	0.98	0.92
1145	0.61	0.29	0.90	0.86
1530	0.47	0.22	0.69	0.67
1870	0.39	0.18	0.57	0.57
2235	0.33	0.16	0.49	0.47
2665	0.28	0.13	0.41	0.40
3000	0.25	0.12	0.37	0.37

3 Results and discussion

We have calculated electron impact single ionization cross-sections for 6p, 6s, 5d, 5p and 5s shells of lead atom and found that ionization cross-sections for 5s shell are negligibly small. In case of direct double ionization, contributions from inner shells have also been included in the calculations. It is found that contribution to direct double ionization from ejection of the second electron from 5s shell is insignificant. We have attempted to analyse the single and direct double ionization cross-sections and to identify the inner shells whose ionization leads to Auger effect contributing to double ionization cross-sections.

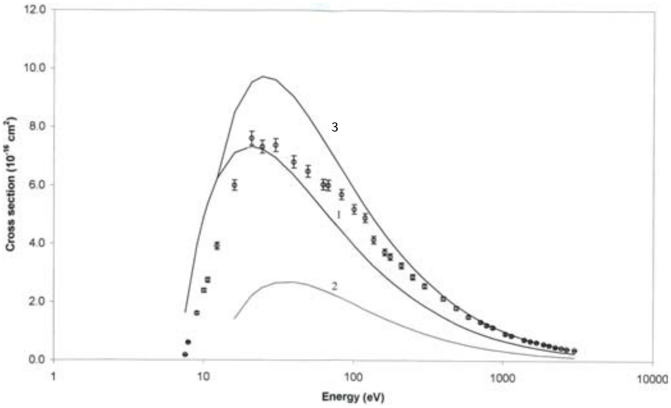

Fig. 1. Electron impact single ionization cross-sections of lead: curve 1, contributions of 6p shell; curve 2, contributions of 6s shell; curve 3, total single ionization cross-sections; Φ experimental data (McCartney et al. [3]).

Quantum mechanical calculations of single ionization cross-sections for a large number of atoms/ions are available in literature. Unfortunately such calculations for lead atom have not been reported so far probably due to complexity of the problem. This is the reason why McCartney et al. have compared their experimental data with the results obtained by Lotz formula [9] and semi-classical expression of Margreiter et al. [10] in limited energy range 15−200 eV. Our calculated results of single ionization cross-sections considering the contributions of 6p and 6s shells only along with the experimental data in the energy range 7.6−3000 eV have been presented in Figure 1 and Table 1. It is found that ionization cross-sections of 6s shell are much smaller as compared to those of 6p shell. The theoretical results of single ionization overestimate the cross-sections in the energy region close to threshold but become within a factor of two of the experimental data beyond 10.64 eV impact energy. The calculated values go on improving with increase in energy and are found to be within a factor of 1.5 beyond 15.94 eV. The agreement of our results becomes better and better with increasing energy and it is found that the values are within a factor of 1.25 of the experimental results beyond 67.5 eV. The gradually improving trend continues at higher energies also and it is remarkable that the theoretical values are in very close agreement with experimental data in the energy region 1145−3000 eV. The peaks in the experimental and theoretical cross-sections are found at 20.74 eV and 24.44 eV impact energies with the magnitudes 7.61×10^{-16} cm^2 and 9.72×10^{-16} cm^2

Table 2. Electron impact double ionization cross-sections of Pb in units of 10^{-16} cm^2.

Energy (eV)	Contributions of direct double ionization	Contributions of $5d$ single ionization	Contributions of $5p$ single ionization	Total	Experiment [3]
22.84	0.04	-	-	0.04	0.004
24.44	0.06	-	-	0.06	0.045
25.94	0.09	-	-	0.09	0.10
34.44	0.43	0.22	-	0.65	0.56
36.94	0.50	0.51	-	1.01	0.66
49.44	0.67	1.31	-	1.98	1.20
67.54	1.00	1.69	-	2.69	1.65
87.14	1.02	1.78	-	2.80	1.91
100.74	0.96	1.78	0.02	2.76	1.88
209.54	0.79	1.47	0.18	2.44	1.57
229.54	0.74	1.41	0.19	2.34	1.52
274.64	0.63	1.30	0.19	2.12	1.42
399.44	0.41	1.06	0.17	1.64	1.24
534.44	0.27	0.89	0.16	1.32	1.02
589.44	0.24	0.83	0.15	1.22	1.00
859.44	0.15	0.64	0.12	0.91	0.78
1045	0.11	0.56	0.11	0.78	0.66
1530	0.06	0.41	0.08	0.55	0.50
1870	0.04	0.35	0.07	0.46	0.44
2235	0.04	0.30	0.06	0.40	0.37
2665	0.03	0.26	0.06	0.35	0.32
3000	0.02	0.24	0.05	0.31	0.30

respectively. The ratio of the magnitudes of the theoretical and experimental peaks is 1.28 and the position of the calculated peak is shifted slightly to higher energy side.

Considering the possibility of vacancy in $5d$ and $5p$ shells to result in Auger effect, we have not included the contributions of these shells to single ionization cross-sections. However, we would like to examine the effects of including these contributions in estimation of single ionization cross-sections. It is found that inclusion of contribution of ionization of $5d$ and $5p$ shells makes the agreement of single ionization cross-sections worse with the experimental data. The ionization cross-sections for $5d$ and $5p$ shells can be seen in Table 2. It is found that the theoretical cross-sections differ from the experimental data by a factor more than 1.6 in the energy range 209.5−859.4 eV and this ratio exceeds 1.75 in the energy region 1045−3000 eV. This is contrary to the usual trend that electron impact single ionization cross-sections for atoms/ions calculated in the BEA show closer agreement with experiment at increasing incident energies. Thus it is reasonable to think that ionization of $5d$ and $5p$ shells leads to Auger effect contributing to double ionization cross-section. We will bring out this idea more clearly in discussion on results of double ionization.

McCartney et al. [3] have shown that their experimental data exhibit surprisingly good agreement with the calculations using the well-known Lotz formula which includes only the direct ionization and excludes contributions from any autoionization process in the energy range 15−200 eV. At the same time they have observed that the single ionization cross-sections based on the semi-classical approach of Margreiter et al. [10] show a peak which is shifted to higher energy than that observed experimentally. In the present work the shift in the position of the cross-section peak has been found to be 3.7 eV only which is much less than that obtained by the semi-classical approach. The prominent structure in the low energy section of the experimental single ionization cross-section curve between 7.6 and 9 eV has been interpreted to be consistent with the known autoionization transition. Unfortunately it is not possible to take into account the effect of these autoionization processes and explain the structure by calculations using the BEA. However, as discussed earlier our calculations show continuously improving agreement with the experiment beyond 10.64 eV impact energy. Thus we find that our calculated single ionization cross-sections agree well with the experimental data throughout the energy range investigated, excepting the energy region close to threshold (7.56−10.64 eV).

The theoretical results of double ionization cross-section along with the experimental data obtained by McCartney et al. have been presented in Figure 2 and Table 2. The calculated results presented in the figure show that the process of direct double ionization from ejection of ($6p$, $6p$), ($6p$, $6s$), ($6p$, $5d$) and ($6p$, $5p$) electrons starts at increasing incident energies corresponding to the respective thresholds for double ionization. However, the direct double ionization cross-sections based on the above mentioned contributions are much smaller as compared to the experimental values. The contributions of Auger effect to double ionization resulting from vacancy in $5d$ and $5p$ shells have been considered in our calculations.

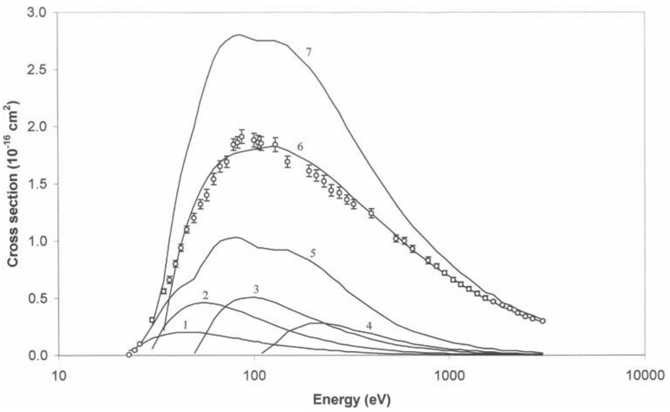

Fig. 2. Electron impact double ionization cross-sections of lead: curves 1, 2, 3 and 4 denote contributions from ejection of ($6p$, $6p$), ($6p$, $6s$), ($6p$, $5d$) and ($6p$, $5p$) electrons respectively; curve 5, direct double ionization cross-sections; curve 6, contributions of the Auger effect; curve 7, total double ionization cross-sections; Φ experimental data (McCartney et al. [3]).

Keeping in view large contributions of Auger emission and non-availability of the Auger yield in the literature, we have assumed the Auger yields for both shells to be unity. First, we would like to discuss detailed comparison of direct double ionization cross-sections with the experimental data. At incident energies 22.8 eV and 24.4 eV the calculated results overestimate the cross-sections but beyond 25.9 eV impact energy they are found to be smaller than the experimental values. At incident energies 229.5 eV, 399.4 eV, 589.4 eV, 859.4 eV and 1045 eV the calculated direct double ionization cross-sections become less than half, one-third, one-fourth, one-fifth and one-sixth of the experimental cross-sections respectively. Beyond 1870 eV the calculated results are found to be negligibly small as compared to experimental values. Such trend of direct double ionization cross-sections strongly supports the idea of the contribution of Auger effect to double ionization cross-sections. As mentioned earlier, single ionization cross-sections for $5s$ shell are negligibly small. In this situation we have considered the contributions of Auger effect due to vacancy in $5d$ and $5p$ shells only.

Now we will examine the double ionization cross-sections including contributions from Auger effect. The processes of Auger emission due to vacancy in $5d$ and $5p$ shells start at 34.4 eV and 100.7 eV impact energies respectively. Due to onset of Auger emission at 34.4 eV it is interesting to note that the contribution of the Auger effect become more than that of direct double ionization at 36.9 eV impact energy and this trend continues at higher energies also. It is surprising that the contributions of the Auger effect to double ionization cross-sections show excellent agreement with experiment at incident energies beyond 36.9 eV (see Fig. 2 and Tab. 2). At first sight one may be tempted to think that the process of the Auger emission alone explains the experimental double ionization cross-sections. Here we would like to mention that calculations of ionization cross-sections in the BEA are not expected to be so accurate as to give results in such a good agreement with experiment. Moreover, lead atom has a good number of weakly bound electrons and therefore one would definitely expect substantial contributions from direct double ionization process. Thus, it is concluded that the excellent agreement of the theoretical results based on the Auger effect is accidental. Our calculated values of total double ionization cross-sections remain within a factor of 2 up to 274.6 eV impact energy. Beyond this energy the agreement of our calculated results improves and the theoretical values are found to be within a factor of 1.5 of the experimental cross-sections up to 534.4 eV. The improving trend of our cross-sections continues at higher energies also and it is found that in the energy range 589.4−3000 eV the calculated cross-sections remain within a factor of 1.25 of the experimental data. It can be seen that the theoretical cross-sections and the experimental results are in very close agreement in the energy range 1870−3000 eV. In this energy region the direct double ionization cross-sections are insignificant and the main contribution to double ionization comes from the Auger effect. The peaks which appear at the same impact energy 87.1 eV in our calculations and experiment are of magnitudes 2.80×10^{-16} cm^2 and 1.91×10^{-16} cm^2 respectively. It is remarkable that the position of the predicted cross-section peak agrees well with the experiment. It is apparent that the calculated results are in satisfactory agreement with the experimental observations throughout the energy range investigated. In particular, our double ionization cross-sections show good agreement with the experimental data in the energy region 589.4−3000 eV being always within a factor of 1.25 of the experimental values.

McCartney et al. [3] have compared their experimental data on double ionization with the results calculated by scaling law proposed by Fisher et al. [11]. This scaling procedure has been discussed by Jha and Roy [19]. In the present case it is seen that the calculations based on scaling law of Fisher et al. do not show satisfactory agreement with the experiment in energy region beyond 100 eV. From the discussion given above we find that the approach adopted in the present work has been successful in predicting satisfactory results of electron impact single and double ionization cross-sections of lead. At the same time the inner shells whose ionization leads to main contribution from Auger effect have been identified.

4 Conclusions

On the basis of the present calculations it is concluded that electron impact single ionization cross-sections of lead in the energy region 10.64−3000 eV are well explained by considering ionization of $6p$ and $6s$ shells only. The calculated direct double ionization cross-sections can not explain the experimental observations. Beyond 36.94 eV impact energy the calculated results are found to be of decreasing importance as compared to contributions from Auger effect and they become insignificant in the energy range 1870−3000 eV. Inclusion of contributions of Auger effect due to vacancy in $5d$ and $5p$ shells brings the theoretical results of double ionization cross-sections in reasonably good agreement with the experimental data. The identification of the above mentioned shells which has been substantiated theoretically is an interesting feature of the present calculations. More elaborate theoretical work is needed for the quantitative understanding of the problem, particularly at low incident energies. It is expected that this work will stimulate other theoretical workers to take up further study of the problem.

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